Topic 1: Geometry: Review, Half-plane Intersection, Adaptive Simpson **Handout**
CS 41100 - CP3 Competitive Programming III (Spring 2025) Purdue University
Instructor: Zhongtang Luo **Date:** January 19, 2025

Sample Problem: Birthday Cake
Link: `https://vjudge.net/problem/Gym-104114B`

Sample Problem: Panda Reserve
Link: `https://vjudge.net/problem/Gym-498660B`

Sample Problem: Art Gallery
Link: `https://vjudge.net/problem/Baekjoon-3800`

Sample Problem: Garden of Thorns
Link: `https://naq23.kattis.com/contests/naq23-fall/problems/naq23.gardenofthorns`

Sample Problem: Military Maneuver
Link: `https://vjudge.net/problem/Gym-104869G`

Birthday Cake

How exciting! Today is your little brother's birthday! That's why you ordered a huge (1×1)-meter cake. It is a special vanilla cake with n sweet chocolate chips and m refreshing strawberries.

You show him your awesome surprise, and... bummer! It turns out that he hates fruit! "*Of course, how could I have forgotten?*" you say. Nonetheless, he has a sweet tooth for chocolate, so he would be happy if you could cut him a piece of the cake that contains no strawberries. To make him happy, you'd want to give him a piece having as many chocolate chips as possible.

The picture above depicts the example test case.

You can only make one cut along a straight line through the cake, and you are not allowed to cut though either chocolate chips or strawberries. What is the maximum number of chocolate chips that you may give your little brother?

Note: The picture above is for illustration purposes. You should consider both chocolate chips and strawberries to be infinitesimally small.

Input

The first line of the input contains two positive integers n $(1 \leq n \leq 50\,000)$ and m $(1 \leq m \leq 100)$ — the number of chocolate chips and strawberries, respectively.

The i-th of the next $n+m$ lines contains two decimal numbers x_i and y_i, $(0 < x_i, y_i < 1)$, representing the coordinates of the i-th ingredient: the first n of the ingredients are chocolate chips, and the remaining m are strawberries.

All numbers are given with at most 6 decimal places. The locations of all $n + m$ ingredients are distinct.

Output

Output a single non-negative integer c, representing the maximum number of chocolate chips that you can give your little brother after cutting the cake exactly once.

Example

Input

```
5 2
0.2 0.6
0.8 0.6
0.6 0.2
0.1 0.2
0.6 0.8
0.6 0.6
0.5 0.5
```

Output

```
3
```

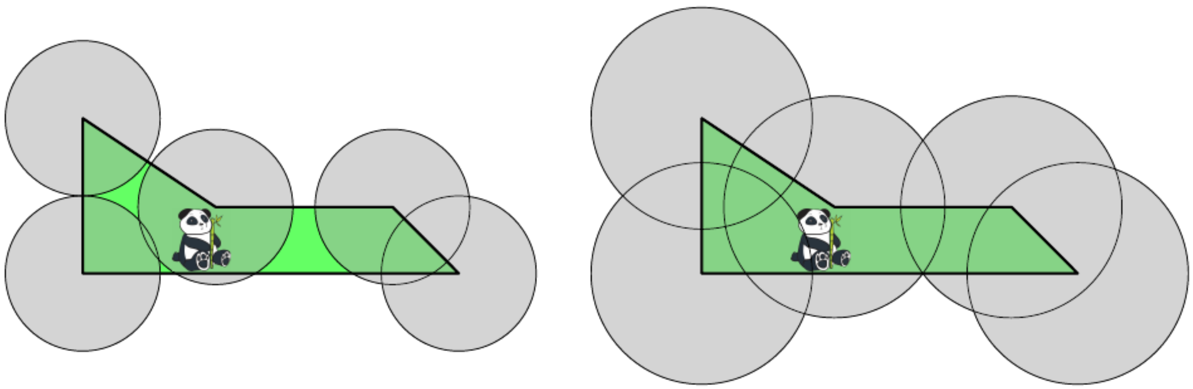
Source

2022 ICPC Southeastern Europe Regional Contest

Panda Reserve

Last month, Sichuan province secured funding to establish the Great Panda National Park, a natural preserve for a population of more than 1800 giant pandas. The park will be surrounded by a polygonal fence. In order for researchers to track the pandas, wireless receivers will be placed at each vertex of the enclosing polygon and each animal will be outfitted with a wireless transmitter. Each wireless receiver will cover a circular area centered at the location of the receiver, and all receivers will have the same range. Naturally, receivers with smaller range are cheaper, so your goal is to determine the smallest possible range that suffices to cover the entire park.

As an example, Figure G.1 shows the park described by the first sample input. Notice that a wireless range of 35 does not suffice (a), while the optimal range of 50 covers the entire park (b).



(a) An insufficient range for covering the park. (b) The minimal range for covering the park.

Figure G.1: Illustration of Sample Input 1.

Input

The first line of the input contains an integer n ($3 \leq n \leq 2000$) specifying the number of vertices of the polygon bounding the park. This is followed by n lines, each containing two integers x and y ($|x|, |y| \leq 10^4$) that give the coordinates (x, y) of the vertices of the polygon in counter-clockwise order. The polygon is simple; that is, its vertices are distinct and no two edges of the polygon intersect or touch, except that consecutive edges touch at their common vertex.

Output

Display the minimum wireless range that suffices to cover the park, with an absolute or relative error of at most 10^{-6}.

Example

Input

```
5
0 0
170 0
140 30
60 30
0 70
```

Output

```
50
```

Input

```
5
0 0
170 0
140 30
60 30
0 100
```

Output

```
51.538820320
```

Input

```
5
0 0
1 2
1 5
0 2
0 1
```

Output

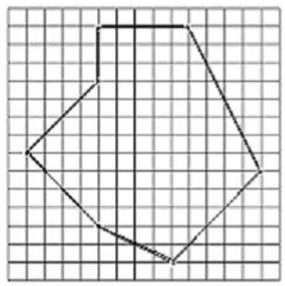
```
1.581138830
```

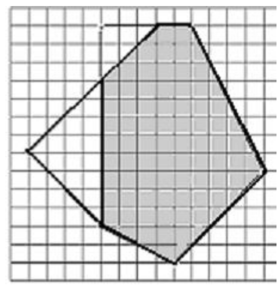
Source

2018 ACM-ICPC World Finals

Art Gallery

The art galleries of the new and very futuristic building of the Center for Balkan Cooperation have the form of polygons (not necessarily convex). When a big exhibition is organized, watching over all of the pictures is a big security concern. Your task is that for a given gallery to write a program which finds the surface of the area of the floor, from which each point on the walls of the gallery is visible. On the figure 1. a map of a gallery is given in some co-ordinate system. The area wanted is shaded on the figure 2.





Input

The number of tasks T that your program have to solve will be on the first row of the input file. Input data for each task start with an integer N, $5 \le N \le 1500$. Each of the next N rows of the input will contain the co-ordinates of a vertex of the polygon — two integers that fit in 16-bit integer type, separated by a single space. Following the row with the co-ordinates of the last vertex for the task comes the line with the number of vertices for the next test and so on.

Output

For each test you must write on one line the required surface — a number with exactly two digits after the decimal point (the number should be rounded to the second digit after the decimal point).

Example

Input

```
1
7
0 0
4 4
4 7
9 7
13 -1
8 -6
4 -4
```

Output

```
80.00
```

Source

Southeastern Europe 2002

Garden of Thorns

Eddy owns a rectangular garden and has noticed some trespassers stomping through his garden. There are some plants that he wants to protect. He hires an assistant, Zyra, to patrol and protect his garden.

Zyra cannot be bothered to monitor his garden, so she plants a circle of thorns centered at a randomly chosen location within the boundaries of his garden. A plant is considered protected if it is strictly inside the circle of thorns - that is, the distance from the plant to the center of the circle of thorns is less than the circle's radius. The circle of thorns may extend outside of the boundary of the rectangular garden, though all plants will be inside or on the boundary of the garden.

Given the random nature of the placement of Zyra's circle of thorns, compute the expected value of the plants that will be protected. Note that Zyra's circle of thorns does not have to be centered at integer coordinates.

Input

The first line of input contains four integers n ($1 \le n \le 10$), r ($1 \le r \le 2000$), w and h ($1 \le w, h \le 1000$), where n is the number of plants in Eddy's garden, r is the radius of Zyra's circle of thorns, w is the width of Eddy's garden and h is the height of the garden.

Each of the next n lines contains three integers x ($0 \le x \le w$), y ($0 \le y \le h$) and v ($1 \le v \le 1000$), where (x, y) denotes the position of a plant from the lower left corner of Eddy's garden, and v is the value of that plant. No two plants will be at the same position.

Output

Output a single real number, which is the expected value of plants which will be protected by Zyra's circle of thorns. Any answer within an absolute or relative error of 10^{-6} will be accepted.

Example

Sample Input 1

```
3 50 100 100
30 10 3
40 10 7
50 90 8
```

Sample Output 1

```
8.4190648693245080380620493087
```

Sample Input 2

```
2 5 3 4
0 0 10
3 4 15
```

Sample Output 2

```
25.0
```

1 Source

North American Qualifier 2023

Military Maneuver

A military maneuver is going on a two-dimensional Cartesian plane, and n enemy targets are hiding somewhere on the battlefield, whose locations are known to our headquarters.

Our headquarters will airdrop a beacon in a rectangular region with sides parallel to the coordinate axes uniformly at random to expose all the enemy targets to our troops on the battlefield so that our troops can surround all the enemy targets. The bottom-left corner of the region is at coordinate (x_l, y_l) while the top-right corner is at coordinate (x_r, y_r).

After being dropped, the beacon will firstly receive two parameters r and R that satisfy $0 \le r \le R$ from our headquarters, then scan an annulus region, that is, the region lying between two concentric circles, where the radius of the inner circle is r and that of the outer circle is R, and finally mark those enemy targets hiding in the scanned region (including the boundary).

However, the beacon can only scan a unit area in a unit of time, and the commander would like to know the expected minimum time for the beacon to scan the designated annulus region so that it can mark all the enemy targets.

Input

The first line contains four integers x_l, y_l, x_r, and y_r ($-10\,000 \le x_l, y_l, x_r, y_r \le 10\,000$, $x_l < x_r$, $y_l < y_r$), denoting the coordinates of the bottom-left and the top-right corners of the rectangular region where the beacon will be dropped.

The second line contains a single integer n ($2 \le n \le 2\,000$), denoting the number of enemy targets on the battlefield.

Each of the following n lines contains two integers x and y ($-10\,000 \le x, y \le 10\,000$), denoting an enemy target located at coordinate (x, y).

It is guaranteed that no two enemy targets share the same locations.

Output

Output a single real number, indicating the expected minimum time for the beacon to scan the designated annulus region.

Your answer is acceptable if its absolute or relative error does not exceed 10^{-6}. Formally speaking, suppose that your output is a and the jury's answer is b, your output is accepted if and only if $\frac{|a-b|}{\max(1,|b|)} \le 10^{-6}$.

Example

Input

```
0 0 2 2
2
3 1
1 3
```

Output

```
8.377580409572781970
```

Input

```
0 0 2 2
2
5 1
1 3
```

Output

```
37.699111843077518863
```

Note

In the first sample case, if the beacon is dropped to $(0.5, 1.5)$, the minimum time as well as the minimum area of the feasible annulus region is 4π. The expected minimum time when the beacon dropped in the rectangular region uniformly at random is $\frac{3}{8}\pi$.

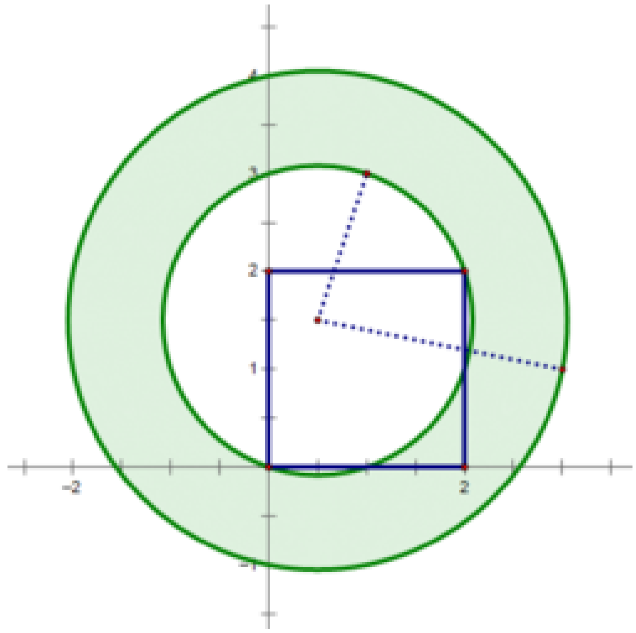


Figure: The feasible annulus region for the beacon at $(0.5, 1.5)$

Source

The 2023 ICPC Asia Shenyang Regional Contest (The 2nd Universal Cup. Stage 13: Shenyang)