Topic 6: Game Theory: SG Function, Search

CS 41100, CP3 Competitive Programming III (Spring 2025) Instructor: Zhongtang Luo
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Learning Objectives

The students will be able to…

1. describe the **game tree** of a strategy game;
2. identify **losing states** and **winning states** of a strategy game;
3. conduct an exhaustive search using the **reverse-topological order** to determine losing states and winning states;
4. describe the **Sprague–Grundy theorem**;
5. compute the **Sprague–Grundy number** of a Nim game;
6. analyze the **Sprague–Grundy number** of a state in an impartial game;

Sample Problems

Problem Name: Endgame
Link: `https://vjudge.net/problem/Baekjoon-19350`

Problem Name: Nim or not Nim?
Link: `http://acm.hdu.edu.cn/showproblem.php?pid=3032`

Problem Name: Sequential Nim
Link: `https://codeforces.com/problemset/problem/1382/B`

Problem Name: Nim
Link: `https://codeforces.com/problemset/problem/2043/F`

Endgame

The game of chess is almost finished. On the chessboard, apart from White and Black kings, there is only a White rook.

You are playing White, and it is your move. Determine the minimal number of moves you need to give a checkmate, provided that your opponent plays optimally and delays his inevitable defeat for as long as possible.

There is a compilation of chess rules at the end of this statement. If you already know them, rest assured: your puny chess skills will not help you solve this problem.

Input

The first line of input contains the number of test cases z ($1 \le z \le 10$). The descriptions of the test cases follow.

Each test case is given on eight lines describing a chessboard. Each of these lines describes a single row and contains exactly eight characters: '.' denotes an empty field, 'W' is the White king, 'B' is the Black king, and 'R' is the White rook. There is exactly one piece of each kind. The starting position is guaranteed to be valid: in particular, kings are not adjacent to each other, and the Black king is not under attack.

There is an empty line after each test case.

Output

For each test case, output a line containing a single integer: the maximal possible number of moves White needs to give a checkmate (per common tradition, count only your moves, not Black's).

Examples

Input

```
2
........
........
........
........
........
.......W
R.......
.......B

....B...
........
..W.....
........
.....R..
........
........
........
```

Output

```
1
2
```

Note

Chess rules:

1. The players alternately move one piece per turn.
2. A player cannot "pass"; on each turn, they have to make a legal move.

3. The king moves one square in any direction (horizontally, vertically, or diagonally).
4. The rook can move any number of squares along any row or column, but may not leap over other pieces.
5. A king is under attack if it is within move range of an opposing piece.
6. A player may not make any move that would put or leave his or her king under attack (in particular, the king cannot be moved to a square adjacent to other king).
7. A Black king can, however, move to a square occupied by the White rook, if the White king is not adjacent to the rook. The rook is then captured and the game ends in a draw.
8. If Black player has no legal move, the game is over; it is either a checkmate (White wins) if the Black king is under attack, or a stalemate (a draw) if it is not.
9. It is known that, in the situation described above (king and rook vs. king), a checkmate is always possible in less than 50 moves.

Source

Petrozavodsk Programming Camp, Winter 2017, Day 1: Jagiellonian U Contest

Nim or not Nim?

Nim is a two-player mathematic game of strategy in which players take turns removing objects from distinct heaps. On each turn, a player must remove at least one object, and may remove any number of objects provided they all come from the same heap.

Nim is usually played as a misere game, in which the player to take the last object loses. Nim can also be played as a normal play game, which means that the person who makes the last move (i.e., who takes the last object) wins. This is called normal play because most games follow this convention, even though Nim usually does not.

Alice and Bob are tired of playing Nim under the standard rule, so they make a difference by also allowing the player to separate one of the heaps into two smaller ones. That is, each turn the player may either remove any number of objects from a heap or separate a heap into two smaller ones, and the one who takes the last object wins.

Input

Input contains multiple test cases. The first line is an integer $1 \le T \le 100$, the number of test cases. Each case begins with an integer N, indicating the number of the heaps, the next line contains N integers $s[0], s[1], \dots, s[N-1]$, representing heaps with $s[0], s[1], \dots, s[N-1]$ objects respectively. ($1 \le N \le 10^6$, $1 \le S[i] \le 2^{31} - 1$)

Output

For each test case, output a line which contains either "Alice" or "Bob", which is the winner of this game. Alice will play first. You may assume they never make mistakes.

Examples

Input

```
2
3
2 2 3
2
3 3
```

Output

```
Alice
Bob
```

Source

2009 Multi-University Training Contest 13 - Host by HIT

Sequential Nim

There are n piles of stones, where the i-th pile has a_i stones. Two people play a game, where they take alternating turns removing stones.

In a move, a player may remove a positive number of stones from the first non-empty pile (the pile with the minimal index, that has at least one stone). The first player who cannot make a move (because all piles are empty) loses the game. If both players play optimally, determine the winner of the game.

Input

The first line contains a single integer t ($1 \le t \le 1000$) — the number of test cases. Next $2t$ lines contain descriptions of test cases.

The first line of each test case contains a single integer n ($1 \le n \le 10^5$) — the number of piles.

The second line of each test case contains n integers $a_1, \dots, a_n$ ($1 \le a_i \le 10^9$) — a_i is equal to the number of stones in the i-th pile.

It is guaranteed that the sum of n for all test cases does not exceed 10^5.

Output

For each test case, if the player who makes the first move will win, output "First". Otherwise, output "Second".

Examples

Input

```
7
3
2 5 4
8
1 1 1 1 1 1 1 1
6
1 2 3 4 5 6
6
1 1 2 1 2 2
1
1000000000
5
1 2 2 1 1
3
1 1 1
```

Output

```
First
Second
Second
First
First
Second
First
```

Note

In the first test case, the first player will win the game. His winning strategy is:

1. The first player should take the stones from the first pile. He will take 1 stone. The numbers of stones in piles will be $[1, 5, 4]$.
2. The second player should take the stones from the first pile. He will take 1 stone because he can't take any other number of stones. The numbers of stones in piles will be $[0, 5, 4]$.
3. The first player should take the stones from the second pile because the first pile is empty. He will take 4 stones. The numbers of stones in piles will be $[0, 1, 4]$.
4. The second player should take the stones from the second pile because the first pile is empty. He will take 1 stone because he can't take any other number of stones. The numbers of stones in piles will be $[0, 0, 4]$.
5. The first player should take the stones from the third pile because the first and second piles are empty. He will take 4 stones. The numbers of stones in piles will be $[0, 0, 0]$.
6. The second player will lose the game because all piles will be empty.

Source

Codeforces Round 658 (Div. 2), Problem B

Nim

Recall the rules of the game "Nim". There are n piles of stones, where the i-th pile initially contains some number of stones. Two players take turns choosing a non-empty pile and removing any positive (strictly greater than 0) number of stones from it. The player unable to make a move loses the game.

You are given an array a, consisting of n integers. Artem and Ruslan decided to play Nim on segments of this array. Each of the q rounds is defined by a segment (l_i, r_i), where the elements $a_{l_i}, a_{l_i+1}, \dots, a_{r_i}$ represent the sizes of the piles of stones.

Before the game starts, Ruslan can remove any number of piles from the chosen segment. However, at least one pile must remain, so in a single round he can remove at most $(r_i - l_i)$ piles. He is allowed to remove 0 piles. After the removal, the game is played on the remaining piles within the segment.

All rounds are independent: the changes made in one round do not affect the original array or any other rounds.

Ruslan wants to remove as many piles as possible so that Artem, who always makes the first move, loses.

For each round, determine:

1. the maximum number of piles Ruslan can remove;
2. the number of ways to choose the maximum number of piles for removal.

Two ways are considered different if there exists an index i such that the pile at index i is removed in one way but not in the other. Since the number of ways can be large, output it modulo 998 244 353.

If Ruslan cannot ensure Artem's loss in a particular round, output -1 for that round.

Input

The first line of input contains two integers n and q ($1 \le n, q \le 10^5$) — the size of the array and the number of segments for which the answers need to be calculated.

The second line of input contains n integers $a_1, a_2, \dots, a_n$ ($0 \le a_i \le 50$) — the elements of the initial array.

The i-th of the next q lines contains two integers l_i, r_i ($1 \le l_i \le r_i \le n$) — the bounds of the segment on which the boys want to play the game during the i-th round.

Output

For each round:

- if Ruslan can win, print two integers — the maximum number of piles that can be removed, and the number of ways to remove the maximum number of piles, taken modulo 998 244 353;
- otherwise print -1.

Examples

Input

```
9 5
0 1 2 1 3 4 5 6 0
1 5
2 5
3 5
4 5
1 9
```

Output

```
4 1
2 1
0 1
-1
8 2
```

Source

Educational Codeforces Round 173 (Rated for Div. 2), Problem F