

# Topic 8: Fast Fourier Transform

CS 41100, CP3 Competitive Programming III (Spring 2025)  
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## Learning Objectives

The students will be able to...

1. describe the input and the output of the **fast Fourier transform** and the **number theoretic transform** algorithm;
2. use the idea of **generating functions** to format problems into polynomial multiplications;
3. apply the **fast Fourier transform** algorithm and the **number theoretic transform** algorithm to solve problems.

## Sample Problems

**Problem Name:** Lightsabers (hard)

**Link:** <https://codeforces.com/contest/958/problem/F3>

**Problem Name:** Nikita and Order Statistics

**Link:** <https://codeforces.com/contest/993/problem/E>

## Lightsabers (hard)

There used to be unrest in the Galactic Senate. Several thousand solar systems had declared their intentions to leave the Republic. But fear not! Master Heidi was able to successfully select the Jedi Knights that have restored peace in the galaxy. However, she knows that evil never sleeps and a time may come when she will need to pick another group of Jedi Knights. She wants to be sure she has enough options to do so.

There are  $n$  Jedi Knights, each of them with a lightsaber of one of  $m$  colors. Given a number  $k$ , compute the number of differently colored collections of  $k$  lightsabers that some  $k$  Jedi Knights might have. Jedi Knights with lightsabers of the same color are indistinguishable (it's not the person, it's the lightsaber color that matters!), and their order does not matter; that is, we consider two collections of Jedi Knights to be different if and only if their vectors of counts of lightsabers of each color are different. We count all subsets, not only contiguous subsegments of the input sequence. Output the answer modulo 1009.

### Input

The first line of the input contains  $n$  ( $1 \leq n \leq 2 \cdot 10^5$ ),  $m$  ( $1 \leq m \leq n$ ) and  $k$  ( $1 \leq k \leq n$ ). The second line contains  $n$  integers in the range  $\{1, 2, \dots, m\}$  representing colors of the lightsabers of subsequent Jedi Knights.

### Output

Output one number: the number of differently colored collections of  $k$  lightsabers modulo 1009.

### Examples

#### Input

```
4 3 2
1 2 3 2
```

#### Output

```
4
```

### Source

Helvetic Coding Contest 2018

## Nikita and Order Statistics

Nikita enjoys solving problems involving order statistics, such as finding the  $k$ -th smallest number in a segment of an array. Currently, Nikita is curious about how many segments of an array exist where a specific number  $x$  is the  $k$ -th smallest number. Essentially, you need to determine the number of segments in a given array for which there are exactly  $k$  numbers less than  $x$  within that segment.

Nikita wants to know the answers to this question for every  $k$  ranging from 0 to  $n$ , where  $n$  is the size of the array.

### Input

The input consists of two lines. The first line contains two integers  $n$  and  $x$  ( $1 \leq n \leq 2 \times 10^5$ ,  $-10^9 \leq x \leq 10^9$ ).

The second line contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $-10^9 \leq a_i \leq 10^9$ ), representing the given array.

### Output

Output  $n + 1$  integers, where the  $i$ -th number corresponds to the answer to Nikita's question for  $k = i - 1$ .

### Examples

#### Input

```
5 3
1 2 3 4 5
```

#### Output

```
6 5 4 0 0 0
```

#### Input

```
2 6
-5 9
```

#### Output

```
1 2 0
```

#### Input

```
6 99
-1 -1 -1 -1 -1 -1
```

#### Output

```
0 6 5 4 3 2 1
```

### Source

Codeforces Round 894 (Div. 3)